NUMERICAL STUDY OF TEMPERATURE FIELDS IN THE CONTINUOUS TEEMING OF STEEL

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A numerical solution is obtained for a Stefan problem modeling the continuous teeming of steel in different regimes.

The process of solidification of molten metal has a significant effect on the initial characteristics of the ingot [1, 2]. Investigators have now discovered the basic laws governing the formation of a continuous-cast ingot in different teeming regimes, and a great deal of experimental data has been accumulated. However, a detailed quantitative investigation of thermal processes taking place during continuous casting requires the use of mathematical modeling and computer technology [3].

Here we mathematically model the thermophysical process of the formation of a continuous-cast ingot. Using dimensionless parameters and constant (averaged) thermophysical characteristics for the melt and ingot, we succeeded in using a fairly simple mathematical model to determine the effect of withdrawal rate, heat of phase transformation, and thermal boundary conditions on ingot quality. We will use the "parabolic" approximation [4] to approximately solve a quasisteady Stefan problem. In this approximation, we ignore the effect of heat flows along the ingot. The computing algorithm is based on the use of a through computing scheme, with smoothing of the coefficients [5].

Formulation of the Problem. We will examine the principal thermal processes taking place in the formation of a continuous-cast cylindrical steel ingot (Fig. 1). Molten metal is poured into the mold, and the metal cools and solidifies. The ingot is withdrawn from the mold at a constant rate. The interest in studying the melt–solid boundary stems from the fact that it in large part determines the quality of the ingot.

The problem is described by the following heat-conduction equation in cylindrical variables:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)=cv\frac{\partial T}{\partial z},\quad 0<r<R,\ 0<z<\infty. \tag{1}$$

The thermal conductivity and heat capacity of the ingot are constant. At the phase boundary S, where $T = T^*$, the temperature is continuous but the heat flux is discontinuous. The magnitude of this discontinuity is equal to the product of the heat of phase transformation and the normal component of the velocity of the phase boundary:

$$(T)_1=(T)_2=T^*,\quad (r,\ z)\in S, \tag{2}$$

$$\left(k\frac{\partial T}{\partial n}\right)_1-\left(k\frac{\partial T}{\partial n}\right)_2=-\lambda v\cos(n,\ z),\quad (r,\ z)\in S. \tag{3}$$

In (3), n is a normal to S, this being an exterior normal relative to the region of the melt 1; cos(n, z) is the cosine of the angle between the normal and the OZ axis (Fig. 1).

We supplement Eq. (1) with the corresponding boundary conditions:

at $z = 0$

$$T(r,\ 0)=T_0, \tag{4}$$

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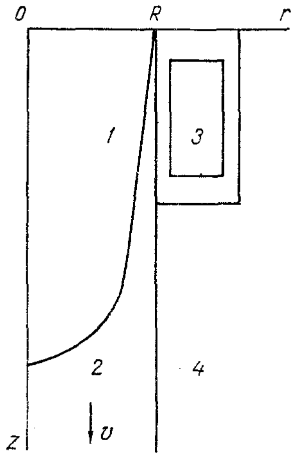

Fig. 1

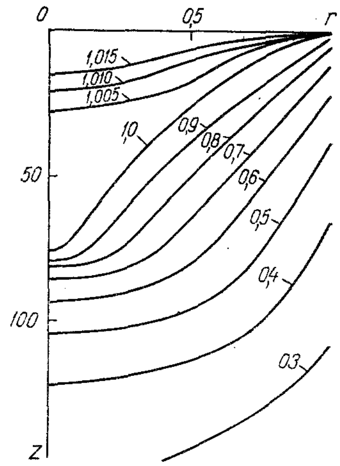

Fig. 2

Fig. 1. Diagram of continuous casting of the steel: 1) melt; 2) solidified part of ingot; 3) mold; 4) secondary cooling zone.

Fig. 2. Isotherms for the base variant.

at $z \to \infty$

$$T(r, z) = T_c, \tag{5}$$

at $r \to 0$

$$r \frac{\partial T}{\partial r}(r, z) \to 0, \tag{6}$$

at $r = R$

$$k \frac{\partial T}{\partial r}(r, z) = -\alpha (T - T_c), \tag{7}$$

where $\alpha = \alpha(z)$ is an assigned function.

Quasisteady Stefan problem (1-7) is not convenient for study. The difficulties arise mainly from the fact that it is necessary to examine the problem in an infinite half-strip (0, R)(0, ∞). We therefore use the "parabolic" approximation (see [4], for example). With sufficiently high withdrawal rates v, we can ignore temperature nonuniformity along the ingot in problem (1-7). To substantiate such an approximation, it will suffice to evaluate the heat-flux relations $W_v = cvT$ and $W_k = k(\partial T/\partial z)$, accounting for heat transfer and thermal conductivity along the ingot. At realistic withdrawal speeds, the temperature field of a continuous-cast steel ingot is characterized by the fact that $q = W_v/W_k \gg 1$, such as $q \sim 10^4$. This allows us to discard the second term in the left side of heat-conduction equation (1).

After introducing dimensionless variables into problem (1-7), with allowance for the above approximation we arrive at the following parabolic problem:

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) = \mathrm{Pe} \frac{\partial u}{\partial z}, \quad 0 < r < 1, \quad z > 0, \tag{8}$$

$$(u)_1 = (u)_2 = 1, \quad (r, z) \in S, \tag{9}$$

$$\left(\frac{\partial u}{\partial n}\right)_1 - \left(\frac{\partial u}{\partial n}\right)_2 = -\mathrm{Pe}\,\mathrm{St} \cos(n, z), \quad (r, z) \in S, \tag{10}$$

$$u(r, 0) = u_0, \tag{11}$$

$$r \frac{\partial u}{\partial r} \to 0, \quad r \to 0, \tag{12}$$

$$\frac{\partial u}{\partial r} = -\mathrm{Bi}(u - u_c), \quad r = 1. \tag{13}$$

Problem (8-13) is characterized by the dimensionless numbers Pe and St and the relation Bi(z). We used the following equation for Bi(z) in our calculations:

$$\mathrm{Bi}(z) = (\mathrm{Bi}_0 - \mathrm{Bi}_1)\exp(-\varkappa z) + \mathrm{Bi}_1, \tag{14}$$

so that $\mathrm{Bi}(0) = \mathrm{Bi}_0$ and $\mathrm{Bi}(\infty) = \mathrm{Bi}_1$. Equation (14) makes it possible to model different cooling regimes for steel ingots.

Numerical Method. We used the generalized formulation [5] to approximately solve problem (8-14). Here, Eq. (8) is replaced by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \mathrm{Pe}(1 + \mathrm{St}\,\delta(u-1))\frac{\partial u}{\partial z}, \quad 0<r<1,\ z>0. \tag{15}$$

In this case, conditions (9) and (10) follow from (15). In the numerical realization, instead of problem (11-15) we solve a different problem involving grid smoothing of the δ-function in the right side of (15). To accomplish this, we employed the procedure of layer-by-layer (at each moment of time) selection of a grid analog of the δ-function so that $\delta(u - 1)$ was always "blurred" only at two nodes of the spatial (relative to r) grid. This local smoothing procedure is mandatory in solving Stefan problems which are close to single-phase problems (the temperature in the melt is close to the phase-transformation temperature).

We employed a classical symmetrical difference scheme (weight $\sigma = 0.5$). No additional internal iterations for nonlinearity (on the free phase boundary) were performed. We conducted methodological tests to determine the appropriate meshes of the grids for the radius r and z, the number of nodes, etc.

Results of Calculations. As the main variant we studied problem (8-14) with $u_0 = 1.02$, $u_C = 0.2$, Pe = 150, St = 0.2, $\mathrm{Bi}_0 = 3$, $\kappa = 0$. Figure 2 shows the isotherms for this variant.

Figure 3 shows the phase boundaries with a change in the Stefan number (solid lines). We note that the heat of phase transformation has a significant effect on solidification. The same figure shows the phase boundaries with a change in the boundary regime (Bi_0 in the present case). The other parameters of the problem remain the same as for the base variant.

Figure 4 shows the effect of the Peclet number (withdrawal rate). The depth of the melt increases with an increase in Pe. Here, we also modeled different boundary regimes and we

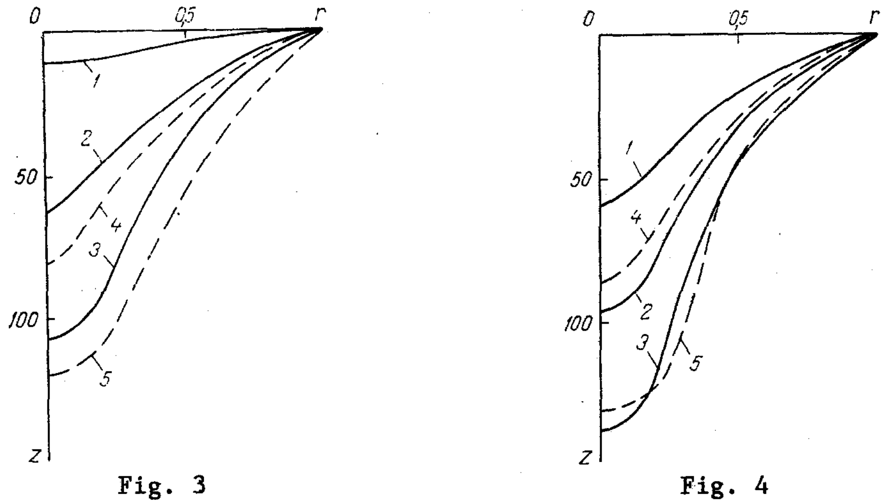

Fig. 3　　　　Fig. 4

Fig. 3. Effect of the Stefan number and the number Bi_0 on the form of the solidification front ($u_0 = 1.02$, $u_C = 0.2$, Pe = 150, $\kappa = 0$): 1) St = 0; 2) 0.15; 3) 0.3 ($\mathrm{Bi}_0 = 3$); 4) $\mathrm{Bi}_0 = 3$; 5) 1 (St = 0.2).

Fig. 4. Form of the solidification front for different values of the Peclet number and the parameter κ ($u_0 = 1.02$, $u_C = 0.2$, $\mathrm{Bi}_0 = 3$, $\mathrm{Bi}_1 = 0.2$): 1) Pe = 100; 2) 150; 3) 200 ($\kappa = 0.025$); 4) $\kappa = 0.01$; 5) 0.06 (Pe = 150).

changed the parameter κ in (14). Similar data was obtained for other values of the determining parameters of the problem.

NOTATION

(r, ϕ, z), cylindrical coordinates; R, radius of the cylindrical ingot; v, withdrawal rate; T^*, phase transformation temperature; T_C, ambient temperature; T_0, initial temperature of the melt; λ, heat of phase transformation; k, thermal conductivity; c, heat capacity; α, coefficient of heat transfer with the environment; Pe = vRc/k, Peclet number; St = λ/cT^*, Stefan number; Bi = $\alpha R/k$, Biot number.

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THE STUDY OF NONLINEAR PROBLEMS OF HIGH-INTENSITY NONSTATIONARY HEAT TRANSFER

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Analytic solutions are found of the nonlinear equations of heat transfer for a dominating effect of relaxation on the thermal flux evolution. The physical interpretation is given of the results obtained as applied to heat exchange problems in one-dimensional regions with moving boundaries.

1. Potential Systems of Equations of Heat Transfer. In the one-dimensional case the equations of heat transfer, in which the finite relaxation time of thermal flux is accounted for, are [1, 2]:

$$cT_t + q_x = 0, \tag{1}$$

$$\lambda T_x + \gamma q_t + q = 0. \tag{2}$$

We take into account that the following inequality is valid in a number of high-intensity nonstationary thermal processes [2-4]

$$|\gamma \partial q/\partial t| \gg q, \quad 0 \leqslant t \leqslant t_1 < \delta < \infty, \quad T \in [T_1, T_2], \tag{3}$$

making it possible to simplify the mathematical model (1), (2) and use in a considered δ-neighborhood of the initial moment of time the approximate equations

$$cT_t + q_x = 0, \quad \lambda T_x + \gamma q_t = 0. \tag{4}$$

The integral equation

$$q = \tau^{-1} \left[q^0(x) - \int_0^t (\tau \lambda T_x/\gamma)\, dt \right], \quad \tau = \exp(t/\gamma),$$

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